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# ONE-LOOP REGGEON-REGGEON-GLUON VERTEX AT ARBITRARY SPACE-TIME DIMENSION *

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**Abstract**

In order to check the compatibility of the gluon Reggeization in QCD with the $s$-channel unitarity, the one-loop correction to the Reggeon-Reggeon-gluon vertex must be known at arbitrary space-time dimension $D$. We obtain this correction from the gluon production amplitude in the multi-Regge kinematics and present an explicit expression for it in terms of a few integrals over the transverse momenta of virtual particles. The one-gluon contribution to the non-forward BFKL kernel at arbitrary $D$ is also obtained.

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# 1 Introduction

Two years ago the kernel of the BFKL equation [1] was obtained in the next-to-leading order (NLO) [2, 3] for the case of forward scattering, i.e. for the momentum transfer $t = 0$ and the singlet colour representation in the $t$-channel. This long awaited event had led to the appearance of a number of papers (see, for instance, [4] and references therein) devoted to the problem of possible applications of the NLO result in the physics of semi-hard processes.

An important way of development of the BFKL approach became the generalization of the obtained results to the non-forward scattering [5]. For singlet colour representation in the $t$-channel, the generalized approach can be used directly for the description of a wide circle of physical processes. The generalization for non-singlet colour states is also of great importance, especially for the antisymmetric colour octet state of two Reggeized gluons in the $t$-channel. This case is especially important since the BFKL approach is based on the gluon Reggeization, although the Reggeization is not proved in the NLO. Therefore, at this order the Reggeization is a hypothesis which must be carefully checked. This can be done using the "bootstrap" equations [5, 6] appearing from the requirement of the compatibility of the gluon Reggeization with the $s$-channel unitarity. In the BFKL approach the scattering amplitude for the high energy process $A+B \to A'+B'$ is presented as the convolution of the impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$, which describe the $A \to A'$ and $B \to B'$ transitions in the particle-Reggeon scattering, and the Green's function $G$ for the Reggeon-Reggeon scattering. The bootstrap conditions for the impact factors are already checked both for the gluon and quark scattering, for helicity conserving and helicity non-conserving amplitudes [7, 8]. We remind that the impact factors are infrared finite [9] for colourless particles only. Considering scattering amplitudes at the parton level, one needs to use an infrared regularization. We use the dimensional regularization, which is commonly adopted. Remarkably, the bootstrap conditions for the impact factors are satisfied at arbitrary space-time dimension $D = 4 + 2\epsilon$.

The kernel of the BFKL equation in the octet channel must also satisfy the bootstrap condition (which is called "first bootstrap condition"). This condition was checked (also for arbitrary $D$) in the part concerning the quark contribution to the kernel [10]. Of course, the most important (and most complicated for verification) is the gluon part of the first bootstrap condition. The gluon part of the kernel in the NLO is expressed in terms of the two-loop gluon trajectory, the one-loop effective vertex for the gluon production in the Reggeon-Reggeon collisions (RRG-vertex) and the two-gluon production contribution. The last contribution was obtained recently [11]. Since the two-loop gluon trajectory [12] and the one-loop RRG-vertex [13] are already known, it seems that the bootstrap condition could be checked. But in the bootstrap equation [5]

$$\frac{g^2 N t}{2(2\pi)^{D-1}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q})^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q})^2} \mathcal{K}^{(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q}) = \omega^{(1)}(t)\, \omega^{(2)}(t) \ , \tag{1.1}$$

where $g$ is the coupling constant, $N$ is the number of colour, $\vec{q}$ is the transverse momentum transfer, $t = -\vec{q}^{\,2}$, $\mathcal{K}^{(8)(1)}$ is the one-loop contribution to the non-forward octet kernel, $\omega^{(1)}$ and $\omega^{(2)}$ are the one- and two-loop contributions to the gluon trajectory, the integration over the transverse momenta $\vec{q}_{1,2}$ is singular. Therefore, in order to check at least the terms finite for $\epsilon \to 0$ in $\omega^{(2)}$ in the R.H.S. of Eq. (1.1), one needs to know the kernel in regions of singularities at arbitrary $\epsilon$ (because, for example, the region of arbitrary small $\vec{q}_1$ does contribute, so that $(\vec{q}_1^{\,2})^\epsilon$ cannot be expanded in powers of $\epsilon$). Unfortunately, the kernel with such accuracy is not known yet, although for the gluon trajectory [12] and for the two-gluon contribution to the octet kernel [11] the integral representations for arbitrary $D$ are known. The problem is in the one-gluon contribution or, in other words, in the RRG-vertex. Let us note that, from now on, we will talk only about the gluon part, i.e. consider pure gluodynamics, since the quark part of the vertex is known at arbitrary $D$ [14]. Instead, in the gluon part firstly only the terms finite for $\epsilon \to 0$ were kept [13], but in the process of calculation of the forward BFKL kernel, it was understood that in this kernel the RRG-vertex at small transverse momenta $\vec{k}$ of the produced gluon must be known at arbitrary $D$. After this, the vertex at small $\vec{k}$ was calculated [15] at arbitrary $D$. Later the results of [13, 15] were obtained by another method in [16]. But for the verification of the bootstrap equation (1.1) it must be known at arbitrary $D$ in a wider kinematical region. Therefore it became clear the necessity of the knowledge of the RRG-vertex at arbitrary $D$, especially taking into account that it can be used not only for the check of the bootstrap, but, for example, in the Odderon problem in the NLO and so on.

In this paper we calculate the RRG-vertex in the NLO and its contribution to the non-forward BFKL kernel at arbitrary $D$. In the next Section we calculate the amplitude of the gluon production in the multi-Regge kinematics for gluon-gluon collisions. The RRG-vertices are defined and calculated in Section 3 and the one-gluon contribution to the BFKL kernel in Section 4.

# 2 The gluon production amplitude

The RRG-vertex can be obtained from the amplitudes of the gluon production in the multi-Regge kinematics (in other words, in the central kinematical region) at collisions of any pair of particles. We will consider the gluon-gluon collisions and use intermediate results obtained in [13] for this process, which are valid at arbitrary $D$. We will use the denotations $p_A$ and $p_B$ ($p_{A'}$ and $p_{B'}$) for the momenta of the incoming (outgoing) gluons, $k$ and $e(k)$ for momentum and polarization vector of the produced gluon; $q_1 = p_A - p_{A'}$ and $q_2 = p_{B'} - p_B$ are the momentum transfers, so that $k = q_1 - q_2$. All the denotations are the same as used in [13], except that for the momentum of the produced gluon, which was called $p_D$ there. The kinematics is defined by the relations

$$s \gg s_1 \sim s_2 \gg |t_1| \sim |t_2| \,, \tag{2.1}$$

where

$$s = (p_A + p_B)^2 \,, \qquad s_1 = (p_{A'} + k)^2 \,, \qquad s_2 = (p_{B'} + k)^2 \,, \qquad t_{1,2} = q_{1,2}^2 \,. \tag{2.2}$$

In terms of the parameters of the Sudakov decomposition

$$k = \beta\, p_A + \alpha\, p_B + k_\perp \,, \qquad\qquad q_i = \beta_i\, p_A + \alpha_i\, p_B + q_{i\perp} \,, \tag{2.3}$$

the relations (2.1) give

$$1 \gg \beta \approx \beta_1 \gg -\alpha_1 \simeq \frac{\vec{q}_1^{\,2}}{s}\,, \qquad 1 \gg \alpha \approx -\alpha_2 \gg \beta_2 \simeq \frac{\vec{q}_2^{\,2}}{s}\,,$$

$$s_1 \approx s\,\alpha\,, \quad s_2 \approx s\,\beta\,, \quad \vec{k}^2 = -k_\perp^2 \approx \frac{s_1 s_2}{s}\,. \tag{2.4}$$

Here and below the vector sign is used for the components of the momenta transverse to the plane of the momenta of the initial particles $p_A$ and $p_B$.

Since we are interested in the RRG-vertex, we will consider the amplitudes with conservation of the helicities of the scattered gluons (helicity non-conservation in the NLO is related with the gluon-gluon-Reggeon vertices only, which are already known at arbitrary $D$ [13]). This amplitude can be presented in the form (cf. [13])

$$A_{2\to 3} = 2s\,g^3\,T^{c_1}_{A'A}\,\frac{1}{t_1}\,T^{d}_{c_2c_1}\,\frac{1}{t_2}\,T^{c_2}_{B'B}\,e^*_\mu(k)\,\mathcal{A}^\mu(q_2,q_1)\,, \tag{2.5}$$

where $T^a_{bc}$ are matrix elements of the colour group generator in the adjoint representation and the amplitude $\mathcal{A}^\mu$ in the Born approximation is equal to $C^\mu(q_2,q_1)$:

$$\mathcal{A}^\mu_{\text{Born}} = C^\mu(q_2,q_1) = -q_{1\perp}^{\;\mu} - q_{2\perp}^{\;\mu} + \frac{p_A^\mu}{s_1}\,(\vec{k}^2 - 2\vec{q}_1^{\,2}) - \frac{p_B^\mu}{s_2}\,(\vec{k}^2 - 2\vec{q}_2^{\,2})\,. \tag{2.6}$$

It was shown in [13] that at one-loop order the amplitude can be presented as the sum of three contributions, which come from integrations over the momenta of the virtual gluons in three different kinematical regions: the region of fragmentation of the particle $A$, the central region and the region of fragmentation of the particle $B$. Correspondingly, we represent $\mathcal{A}^\mu$ as

$$\mathcal{A}^\mu = \mathcal{A}^\mu_{\text{Born}} + \mathcal{A}^\mu_A + \mathcal{A}^\mu_{\text{as}} + \mathcal{A}^\mu_B\,, \tag{2.7}$$

where for the contribution of the fragmentation of the particle $A$, we obtain from Eq. (63) of [13]

$$\mathcal{A}^\mu_A = C^\mu(q_2,q_1)\,\overline{g}^2\,(\vec{q}_1^{\,2})^\epsilon\,\frac{\Gamma^2(1+\epsilon)}{\Gamma(4+2\epsilon)}\,\frac{(-2)}{\epsilon^2}\left(3(1+\epsilon)^2+\epsilon^2\right), \tag{2.8}$$

where $\overline{g}^2 = g^2 N\Gamma(1-\epsilon)/(4\pi)^{2+\epsilon}$. In the derivation of (2.8) from Eq. (63) of [13], we have used the relations

$$e^\alpha_{A\lambda}\,e^{*\alpha'}_{A'\lambda}\left(\delta_{\perp\alpha\alpha'} - (D-2)\frac{q_{1\perp\alpha}\,q_{1\perp\alpha'}}{q_{1\perp}^{\,2}}\right) = 0\,, \qquad e^\alpha_{A\lambda}\,e^{*\alpha'}_{A'\lambda}\,\delta_{\perp\alpha\alpha'} = -1\,, \tag{2.9}$$

for the polarization vectors $e_{A\lambda}$ and $e_{A'\lambda}$ of the particles $A$ and $A'$ with the definite helicity $\lambda$ in the $D$-dimensional space-time. The analogous relations exist for the particle $B$. Evidently, the contribution from the fragmentation region of the particle $B$ can be obtained from (2.8) by the substitution $\vec{q}_1^{\,2} \to \vec{q}_2^{\,2}$.

The contribution of the central region is given by Eqs. (77), (80) and (81) of [13]. At this step in [13] was not yet made an expansion in $\epsilon$. Let us pay attention that in these formulae the anti-symmetrization with respect to the substitution $p_B \longleftrightarrow -p_{B'}$ must be done. The formulae (80) and (81) of [13] can be greatly simplified and the contribution $\mathcal{A}^{\mu}_{\rm as}$, with accuracy up to terms proportional to $k^\mu$ and therefore not contributing to (2.7), can be presented as

$$\mathcal{A}^{\mu}_{\rm as} = -\frac{2g^2N}{(2\pi)^D}\,\widehat{\mathcal{S}}\left\{\left(C^\mu(q_2,q_1)-2\vec{k}^2\mathcal{P}^\mu\right)\left[s_1t_1I_{4A}-\frac{4(D-2)}{t_1}\frac{p_A^\rho p_B^\sigma}{s}I_2^{\rho\sigma}(q_1)-2t_1\frac{p_B^\rho}{s}I_{3A}^\rho\right.\right.$$
$$\left.+4\vec{k}^2\frac{p_A^\rho}{s_1}I_3^\rho-2t_1I_{3A}+\vec{k}^2I_3-2I_2(q_1)\right]+4\mathcal{P}^\mu\left[\frac{s_1s_2}{4}t_1t_2I_5+\frac{s_1t_1}{2}(\vec{k}^2-t_2)I_{4A}-2\vec{k}^2t_1\frac{p_A^\rho}{s_1}I_3^\rho\right.$$
$$\left.-\vec{k}^2t_2I_3-\vec{k}^2\left(2I_2(q_1)+t_1I_{3A}\right)-t_2I_2(q_1)\right]-2\left(g^{\mu\rho}-2\frac{p_B^\mu k^\rho}{s_2}\right)\left[\frac{t_1s_2}{s}I_{3A}^\rho\right.$$
$$\left.+2(D-2)\frac{\vec{k}^2}{t_1}\frac{p_A^\sigma}{s_1}I_2^{\rho\sigma}(q_1)\right]-4(D-2)\frac{p_A^\rho p_B^\sigma}{s}\left[I_3^{\mu\rho\sigma}+I_3^{\mu\rho}k^\sigma-2\frac{p_A^\mu}{s_1}I_2^{\rho\sigma}(q_1)\right]$$
$$\left.-st_1t_2\left[2\frac{p_B^\mu}{s_2}I_4^{(1)}+I_5^\mu\right]+2(\vec{k}^2+t_1+t_2)\left[2\frac{p_A^\mu}{s_1}I_2(q_1)-I_3^\mu\right]\right\}\ , \tag{2.10}$$

where

$$\mathcal{P}^\mu=\frac{p_A^\mu}{s_1}-\frac{p_B^\mu}{s_2} \tag{2.11}$$

and $\widehat{\mathcal{S}}$ is the operator of symmetrization with respect to each of the substitutions $p_A\longleftrightarrow -p_A$ and $p_B\longleftrightarrow -p_B$ and anti-symmetrization with respect to the substitution

$$p_A\longleftrightarrow p_B\ ,\qquad q_1\longleftrightarrow -q_2\ . \tag{2.12}$$

The $I$'s in Eq. (2.10) are the integrals introduced in [13]:

$$\begin{aligned}
I_2^{\mu_1\cdots\mu_n}(q) &= \frac{1}{i}\int d^Dp\,\frac{p^{\mu_1}\cdots p^{\mu_n}}{(p^2+i\varepsilon)[(p+q)^2+i\varepsilon]}\ ,\\
I_3^{\mu_1\cdots\mu_n} &= \frac{1}{i}\int d^Dp\,\frac{p^{\mu_1}\cdots p^{\mu_n}}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+k)^2+i\varepsilon]}\ ,\\
I_{3A}^{\mu_1\cdots\mu_n} &= \frac{1}{i}\int d^Dp\,\frac{p^{\mu_1}\cdots p^{\mu_n}}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+p_A)^2+i\varepsilon]}\ ,\\
I_4^{(1)} &= \frac{1}{i}\int d^Dp\,\frac{1}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+p_A)^2+i\varepsilon][(p-p_B)^2+i\varepsilon]}\ ,\\
I_{4A} &= \frac{1}{i}\int d^Dp\,\frac{1}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+q_2)^2+i\varepsilon][(p+p_A)^2+i\varepsilon]}\ ,\\
I_5 &= \frac{1}{i}\int d^Dp\,\frac{1}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+q_2)^2+i\varepsilon][(p+p_A)^2+i\varepsilon][(p-p_B)^2+i\varepsilon]}\ ,\\
I_5^\mu &= \frac{1}{i}\int d^Dp\,\frac{(p+q_1)^\mu}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+q_2)^2+i\varepsilon][(p+p_A)^2+i\varepsilon][(p-p_B)^2+i\varepsilon]}\ .
\end{aligned} \tag{2.13}$$

In the representation (2.10) the transversality to the momenta $k^\mu$ (which guarantees the gauge invariance of the amplitude) of the first three terms with square brackets is evident, while the transversality of the last three terms can be easily checked with account of the anti-symmetrization (2.12).

The integrals with two or three denominators ($I_2^{\mu_1\cdots\mu_n}$, $I_3^{\mu_1\cdots\mu_n}$, $I_{3A}^{\mu_1\cdots\mu_n}$) were calculated in [13] at arbitrary $D$, whereas for the other integrals the expansion in $\epsilon$ was used. It occurs that, in the multi-Regge kinematics (2.1), the integral $I_4^{(1)}$ can also be calculated at arbitrary $D$, as it is shown in Appendix A. Unfortunately, the integrals $I_{4A}$, $I_5$ and $I_5^\mu$ cannot be expressed in terms of elementary functions at arbitrary $D$. It is possible, however, to perform the integration over the longitudinal variables in the Sudakov decomposition for $p$ and to express them in terms of the same $(D-2)$-dimensional integrals over $\vec{p}$, which appear in the two-gluon contribution to the kernel [11]. This is done in Appendix A, where, for completeness, the integrals with two and three denominators are also given.

The integrals which cannot be expressed through elementary functions, and will be used below, are the following:

$$\mathcal{I}_{4A}=\int_0^1\frac{dx}{1-x}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\left[\frac{x}{[(1-x)\vec{k}_1^2+x(\vec{k}_1+\vec{q}_2)^2](\vec{k}_1-x\vec{k})^2}-\frac{1}{(\vec{k}_1+\vec{q}_2)^2(\vec{k}_1-\vec{k})^2}\right],$$

$$\mathcal{I}_5=\int_0^1\frac{dx}{1-x}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{k}_1^2[(1-x)\vec{k}_1^2+x(\vec{k}_1-\vec{q}_1)^2]}\left(\frac{x^2}{(\vec{k}_1-x\vec{k})^2}-\frac{1}{(\vec{k}_1-\vec{k})^2}\right)\,,$$

$$\mathcal{L}_3=\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{k}_1^2(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_2)^2}\ln\left(\frac{(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_2)^2}{\vec{k}^2\,\vec{k}_1^2}\right)\,,$$

$$\mathcal{I}_3=\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{k}_1^2(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_2)^2}\,, \tag{2.14}$$

as well as $\mathcal{I}_{4B}$, which is obtained from $\mathcal{I}_{4A}$ by the substitution $q_1 \longleftrightarrow -q_2$, and $\mathcal{I}_5^\mu$, $\mathcal{L}_3^\mu$ and $\mathcal{I}_3^\mu$, the first of which differs from the integral $\mathcal{I}_5$ by the factor $(k_1-q_1)_\perp^\mu$ and the other two from $\mathcal{L}_3$ and $\mathcal{I}_3$, respectively, by the factor $k_{1\perp}^{\ \mu}$, in the corresponding integrands.

Using the results of the Appendix A, one gets

$$\mathcal{A}_{\rm as}^\mu=\overline{g}^2\left\{-2t_1t_2\mathcal{F}_5^\mu+r_{\rm as}C^\mu(q_2,q_1)+2t_1r_A\frac{p_A^\mu}{s_1}-2t_2r_B\frac{p_B^\mu}{s_2}\right\}\,, \tag{2.15}$$

where

$$\mathcal{F}_5^\mu=\mathcal{I}_5^\mu+\mathcal{L}_3^\mu+\frac{1}{2}\ln\left(\frac{s(-s)(\vec{k}^2)^2}{s_1(-s_1)s_2(-s_2)}\right)\mathcal{I}_3^\mu\,,$$

$$r_{\rm as}=\left\{\frac{t_1t_2}{2}\mathcal{F}_5+t_2\mathcal{I}_{4B}+\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}_1^{\,2})^\epsilon\left[-\frac{1}{2}\ln\left(\frac{s_1(-s_1)}{t_1^2}\right)+2\psi(\epsilon)-\psi(2\epsilon)-\psi(1-\epsilon)\right.\right.$$
$$+\frac{1}{2\epsilon(1+2\epsilon)(3+2\epsilon)}\left(\frac{t_1(3+14\epsilon+8\epsilon^2)-t_2(3+3\epsilon+\epsilon^2)}{t_1-t_2}\right.$$
$$\left.\left.\left.+\frac{\vec{k}^2t_1\,\epsilon}{(t_1-t_2)^3}\Big((2+\epsilon)t_2-\epsilon\,t_1\Big)\right)\right]\right\}+\Big\{A\longleftrightarrow B\Big\}\,,$$

$$\mathcal{F}_5 = \mathcal{I}_5 - \mathcal{L}_3 - \frac{1}{2}\ln\left(\frac{s(-s)(\vec{k}^2)^2}{s_1(-s_1)s_2(-s_2)}\right)\mathcal{I}_3 \ ,$$

$$r_A = -t_2(t_1\mathcal{F}_5 + \mathcal{I}_{4B}) + \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}_2^{\,2})^\epsilon\left[\frac{1}{2}\ln\left(\frac{s_1(-s_1)s_2(-s_2)}{s(-s)t_2^2}\right) - \psi(1) - \psi(\epsilon)\right.$$

$$\left.+\psi(1-\epsilon) + \psi(2\epsilon)\right] + \frac{\Gamma^2(\epsilon)}{2t_1(1+2\epsilon)\Gamma(2\epsilon)(3+2\epsilon)}\left\{\left((\vec{q}_1^{\,2})^{\epsilon+1}\left[\frac{t_2}{t_1-t_2}(11+7\epsilon)\right.\right.\right.$$

$$\left.\left.\left.+\frac{\vec{k}^2}{(t_1-t_2)^3}\Big(t_2(t_1+t_2) - \epsilon\, t_1(t_1-t_2)\Big) + \frac{(\vec{k}^2)^2}{(t_1-t_2)^3}\Big((2+\epsilon)t_2 - \epsilon\, t_1\Big)\right]\right) + (A \longleftrightarrow B)\right\} \ ,$$

$$r_B = r_A(A \longleftrightarrow B) \ . \tag{2.16}$$

Here and below $A \longleftrightarrow B$ means the substitution (2.12). The amplitude $\mathcal{A}^\mu$ must be anti-symmetric under this substitution. The amplitude (2.15) has this property since $C^\mu(q_2, q_1)$ (Eq. (2.6)) and $\mathcal{F}_5^\mu$ (Eqs. (2.16)) change their sign under this substitution.

Note that in all physical regions we have $s_1 s_2/s = \vec{k}^2$, so that

$$\ln\left(\frac{s(-s)(\vec{k}^2)^2}{(s_1(-s_1)s_2(-s_2))}\right) = i\,\pi \ . \tag{2.17}$$

Below we will work in the physical region of the $s$-channel and use this relation.

The total amplitude $\mathcal{A}^\mu$ is defined by (2.7), (2.8) and (2.15) and can be represented as

$$\mathcal{A}^\mu = C^\mu(q_2, q_1)\left(1 + \bar{g}^2 r\right) + \bar{g}^2\left\{-2t_1t_2\mathcal{F}_5^\mu + 2t_1 r_A\frac{p_A^\mu}{s_1} - 2t_2 r_B\frac{p_B^\mu}{s_2}\right\} \ , \tag{2.18}$$

where

$$r = r_{\rm as} - \frac{2}{\epsilon^2}\frac{\Gamma^2(1+\epsilon)}{\Gamma(4+2\epsilon)}\left(3(1+\epsilon)^2 + \epsilon^2\right)\left((\vec{q}_1^{\,2})^\epsilon + (\vec{q}_2^{\,2})^\epsilon\right) \tag{2.19}$$

and $r_{\rm as}$, $r_A$ and $r_B$ are defined in (2.16). The gauge invariance of the amplitude (2.5) follows from the properties of the amplitudes (2.8) and (2.15):

$$k_\mu\mathcal{A}_A^\mu = k_\mu\mathcal{A}_B^\mu = k_\mu\mathcal{A}_{\rm as}^\mu = 0 \ . \tag{2.20}$$

The first two of these relations follow evidently from the transversality of $C^\mu(q_2, q_1)$ (Eq. (2.6)), whereas the last of them is not so trivial and is fulfilled due to the equality:

$$2\,k_\mu\,\mathcal{F}_5^\mu \equiv f_- = \left[-t_1\mathcal{F}_5 - \mathcal{I}_{4B} - \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}_2^{\,2})^{\epsilon-1}\left(\frac{1}{2}\ln\left(\frac{s_1(-s_1)s_2(-s_2)}{s(-s)t_2^2}\right)\right.\right.$$

$$\left.\left.+\psi(1-\epsilon) - \psi(\epsilon) + \psi(2\epsilon) - \psi(1)\right)\right] - \Big[A \longleftrightarrow B\Big] \ , \tag{2.21}$$

which is derived in the Appendix B. The amplitude can be written in an explicit gauge invariant form if we use an analogous relation, also obtained in the Appendix B:

$$2\,(q_1+q_2)_\mu\,\mathcal{F}_5^\mu \equiv f_+ = \left\{-t_1\mathcal{F}_5 - \mathcal{I}_{4B} - \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}\left[(\vec{q}_2^{\,2})^{\epsilon-1}\left(\frac{1}{2}\ln\left(\frac{s_1(-s_1)s_2(-s_2)}{s(-s)t_2^2}\right)\right.\right.\right.$$

$$+\psi(1-\epsilon)-\psi(\epsilon)+\psi(2\epsilon)-\psi(1)\Big)$$

$$-(\vec{k}^2)^{\epsilon-1}\left(\frac{1}{2}\ln\left(\frac{s_1(-s_1)s_2(-s_2)}{s(-s)(\vec{k}^2)^2}\right)+\psi(1-\epsilon)-\psi(\epsilon)\right)\Bigg]\Bigg\}+\Big\{A\longleftrightarrow B\Big\}\,. \tag{2.22}$$

Since the general form for $\mathcal{F}_5^\mu$ is

$$\mathcal{F}_5^\mu = r_-\,k_\perp^\mu + r_+\,(q_1+q_2)_\perp^\mu\,, \tag{2.23}$$

we can express $r_-$ and $r_+$ in terms of $f_-$ and $f_+$:

$$r_+ = \frac{f_-(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})-f_+\vec{k}^2}{8(\vec{q}_1^{\,2}\vec{q}_2^{\,2}-(\vec{q}_1\vec{q}_2)^2)}\,, \qquad r_- = \frac{f_+(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})-f_-(\vec{q}_1+\vec{q}_2)^2}{8(\vec{q}_1^{\,2}\vec{q}_2^{\,2}-(\vec{q}_1\vec{q}_2)^2)}\,. \tag{2.24}$$

Rewriting then (2.23) as

$$\mathcal{F}_5^\mu = r_-\,k^\mu - r_+\left[C^\mu(q_2,q_1)+2\vec{q}_1\vec{q}_2\left(\frac{p_A^\mu}{s_1}-\frac{p_B^\mu}{s_2}\right)\right]+\frac{f_-}{2}\left(\frac{p_A^\mu}{s_1}+\frac{p_B^\mu}{s_2}\right) \tag{2.25}$$

and using this equality in the expression (2.15) for $\mathcal{A}_{\rm as}^\mu$, we obtain the explicitly gauge invariant form for the amplitude $\mathcal{A}^\mu$:

$$\mathcal{A}^\mu = C^\mu(q_2,q_1)(1+\overline{g}^2 r_C)+\mathcal{P}^\mu\overline{g}^2\,2t_1t_2\,r_{\mathcal{P}}\,, \tag{2.26}$$

where the terms proportional to $k^\mu$ were omitted and

$$r_C = \Bigg\{t_1t_2\left(r_+ + \frac{\mathcal{F}_5}{2}\right)+t_2\mathcal{I}_{4B}+\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}_1^{\,2})^\epsilon\Bigg[-\frac{1}{2}\ln\left(\frac{s_1(-s_1)}{t_1^2}\right)+2\psi(\epsilon)-\psi(2\epsilon)$$

$$-\psi(1-\epsilon)+\frac{1}{2\epsilon(1+2\epsilon)(3+2\epsilon)}\Bigg(-3(1+\epsilon)-\frac{\epsilon^2}{1+\epsilon}+\frac{t_1(3+14\epsilon+8\epsilon^2)-t_2(3+3\epsilon+\epsilon^2)}{t_1-t_2}$$

$$+\frac{\vec{k}^2\,t_1\,\epsilon}{(t_1-t_2)^3}\Big((2+\epsilon)t_2-\epsilon\,t_1\Big)\Bigg)\Bigg]\Bigg\}+\Big\{A\longleftrightarrow B\Big\}\,,$$

$$r_{\mathcal{P}} = \Bigg\{(\vec{q}_1\vec{q}_2)r_+ - \frac{t_1+t_2}{4}\mathcal{F}_5-\frac{\mathcal{I}_{4B}}{2}+\frac{1}{2}\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}_1^{\,2})^{\epsilon-1}\Bigg(-\frac{1}{2}\ln\left(\frac{s_1(-s_1)s_2(-s_2)}{s(-s)t_1^2}\right)+\psi(1)$$

$$+\psi(\epsilon)-\psi(1-\epsilon)-\psi(2\epsilon)+\frac{t_1}{t_2\,(1+2\epsilon)(3+2\epsilon)}\Bigg[\frac{t_2}{t_1-t_2}(11+7\epsilon)$$

$$+\frac{\vec{k}^2}{(t_1-t_2)^3}\Big(t_2(t_1+t_2)-\epsilon\,t_1(t_1-t_2)\Big)+\frac{(\vec{k}^2)^2}{(t_1-t_2)^3}\Big((2+\epsilon)t_2-\epsilon\,t_1\Big)\Bigg]\Bigg)\Bigg\}+\Big\{A\longleftrightarrow B\Big\}\,. \tag{2.27}$$

The functions $r_+$, $\mathcal{F}_5$ and $\mathcal{I}_{4B}$ entering Eq. (2.27) are given by (2.21)-(2.24), (2.16) and (2.14), respectively.

## 3 The Reggeon-Reggeon-gluon vertices

Since the production amplitude must not have simultaneously discontinuities in the overlapping channels $s_1$ and $s_2$, it cannot be a simple generalization of the Regge form for the elastic amplitude. Instead, it has the form [13, 17]

$$8\,g^3\,\mathcal{A}^\mu = \Gamma(t_1;\vec{k}^2)\Gamma(t_2;\vec{k}^2)\left\{\left[\left(\frac{s_1}{\vec{k}^2}\right)^{\omega_1-\omega_2}+\left(\frac{-s_1}{\vec{k}^2}\right)^{\omega_1-\omega_2}\right]\left[\left(\frac{s}{\vec{k}^2}\right)^{\omega_2}+\left(\frac{-s}{\vec{k}^2}\right)^{\omega_2}\right]R^\mu\right.$$

$$\left.+\left[\left(\frac{s_2}{\vec{k}^2}\right)^{\omega_2-\omega_1}+\left(\frac{-s_2}{\vec{k}^2}\right)^{\omega_2-\omega_1}\right]\left[\left(\frac{s}{\vec{k}^2}\right)^{\omega_1}+\left(\frac{-s}{\vec{k}^2}\right)^{\omega_1}\right]L^\mu\right\}\,, \tag{3.1}$$

where $\omega_i=\omega(t_i)$ and we have chosen $\vec{k}^2$ as the scale of energy, since with this choice the one-gluon contribution to the BFKL kernel is expressed through the RRG-vertex in the simplest way [18]. $\Gamma(t_i;\vec{k}^2)$ are the helicity conserving gluon-gluon-Reggeon vertices; they depend on $\vec{k}^2$ as on the energy scale [18]. $R^\mu$ and $L^\mu$ are the right and left RRG-vertices, depending on $\vec{q}_1$ and $\vec{q}_2$. They are real in all physical channels, as well as $\Gamma(t_i;\vec{k}^2)$.

In the one-loop approximation we have

$$\omega(t)=\omega^{(1)}(t)=-\overline{g}^2\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}^{\,2})^\epsilon\,, \tag{3.2}$$

$$\Gamma(t;\vec{k}^2)=g\Big(1+\Gamma^{(1)}(t;\vec{k}^2)\Big)\,, \tag{3.3}$$

where

$$\Gamma^{(1)}(t;\vec{k}^2)=\overline{g}^2\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}^{\,2})^\epsilon\left[\psi(\epsilon)-\frac{1}{2}\psi(1)-\frac{1}{2}\psi(1-\epsilon)\right.$$

$$\left.+\frac{9(1+\epsilon)^2+2}{4(1+\epsilon)(1+2\epsilon)(3+2\epsilon)}-\frac{1}{2}\ln\left(\frac{\vec{k}^2}{\vec{q}^{\,2}}\right)\right]\,. \tag{3.4}$$

In the same approximation we obtain from (3.1)

$$2g\left\{\mathcal{A}^\mu-C^\mu(q_2,q_1)\left[\Gamma^{(1)}(t_1;\vec{k}^2)+\Gamma^{(1)}(t_2;\vec{k}^2)+\frac{\omega_1}{2}\ln\left(\frac{s_1(-s_1)}{(\vec{k}^2)^2}\right)+\frac{\omega_2}{2}\ln\left(\frac{s_2(-s_2)}{(\vec{k}^2)^2}\right)\right.\right.$$

$$\left.\left.+\frac{\omega_1+\omega_2}{4}\ln\left(\frac{s(-s)(\vec{k}^2)^2}{s_1(-s_1)s_2(-s_2)}\right)\right]\right\}=R^\mu+L^\mu+(R^\mu-L^\mu)\frac{\omega_1-\omega_2}{4}\ln\left(\frac{s_1(-s_1)s_2(-s_2)}{s(-s)(\vec{k}^2)^2}\right)\,, \tag{3.5}$$

where now $\omega_i\equiv\omega^{(1)}(t_i)$. Since in the physical region of the $s$-channel the relation (2.17) holds, the combinations $R^\mu+L^\mu$ and $R^\mu-L^\mu$ are determined by the real and imaginary parts of the L.H.S. of Eq. (3.5) in this region. Using (2.18) and comparing the imaginary parts in (3.5), we obtain:

$$R^\mu-L^\mu=\frac{2g\overline{g}^2}{\omega_1-\omega_2}\left\{\left[2t_1t_2\mathcal{I}_3^\mu+\left(C^\mu(q_2,q_1)-4t_1\frac{p_A^\mu}{s_1}\right)\right.\right.$$

$$\times \left( t_1 t_2 \mathcal{I}_3 - \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}_2^{\,2})^\epsilon \right) \Bigg] - [A \longleftrightarrow B] \Bigg\} \ . \tag{3.6}$$

We can express $\mathcal{I}_3^\mu$ in term of $\mathcal{I}_3$ by comparing the imaginary parts of (2.25). In this way we obtain from (3.6) the explicitly gauge invariant expression:

$$R^\mu - L^\mu = \frac{2g\overline{g}^2}{\omega_1 - \omega_2} \Bigg\{ -C^\mu(q_2, q_1) \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{k}^{\,2})^\epsilon + \frac{(\vec{q}_1 \vec{q}_2) C^\mu(q_2, q_1) + 2\vec{q}_1^{\,2} \vec{q}_2^{\,2} \mathcal{P}^\mu}{\vec{q}_1^{\,2} \vec{q}_2^{\,2} - (\vec{q}_1 \vec{q}_2)^2}$$

$$\times \left[ \vec{q}_1^{\,2} \vec{q}_2^{\,2} \vec{k}^{\,2} \mathcal{I}_3 + \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \Big( (\vec{q}_1^{\,2})^\epsilon (\vec{q}_2 \vec{k}) - (\vec{q}_2^{\,2})^\epsilon (\vec{q}_1 \vec{k}) - (\vec{k}^{\,2})^\epsilon (\vec{q}_1 \vec{q}_2) \Big) \right] \Bigg\} \ . \tag{3.7}$$

Evidently, the same result is obtained from the imaginary part of (3.5) if the representation (2.26) for $\mathcal{A}^\mu$ is used.

The integral $\mathcal{I}_3$ (see (2.14)) cannot be expressed through elementary functions at arbitrary $D$. For $\epsilon \to 0$ we have (see [13] or Appendix C)

$$\mathcal{I}_3 \simeq \frac{1}{\epsilon} \left[ \frac{(\vec{q}_1^{\,2} \vec{q}_2^{\,2})^{\epsilon-1}}{(\vec{k}^{\,2})^\epsilon} + \frac{(\vec{q}_1^{\,2} \vec{k}^{\,2})^{\epsilon-1}}{(\vec{q}_2^{\,2})^\epsilon} + \frac{(\vec{q}_2^{\,2} \vec{k}^{\,2})^{\epsilon-1}}{(\vec{q}_1^{\,2})^\epsilon} \right] . \tag{3.8}$$

Using this expression we obtain from (3.7)

$$R^\mu - L^\mu \simeq -\frac{4g\overline{g}^2}{\omega_1 - \omega_2} C^\mu(q_2, q_1) \left( \frac{1}{\epsilon} + (\vec{k}^{\,2})^\epsilon \right) \tag{3.9}$$

for $\epsilon \to 0$, in agreement with [13] (see also [18]).

It's easy to obtain from (3.7) also the limit $\vec{k} \to 0$ at arbitrary $D$. In this limit the main contribution to the integral $\mathcal{I}_3$ (see (2.14)) comes from the region $\vec{k}_1 \simeq \vec{q}_1 \simeq \vec{q}_2$, so that in the integrand the replacement

$$\frac{1}{\vec{k}_1^{\,2}} \to \frac{1}{\vec{Q}^{\,2}} \ , \qquad \vec{Q} = \frac{\vec{q}_1 + \vec{q}_2}{2} \tag{3.10}$$

can be made. After this we have gently

$$\mathcal{I}_3 \simeq \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \frac{(\vec{k}^{\,2})^{\epsilon-1}}{\vec{Q}^{\,2}}, \tag{3.11}$$

and using this result we see that only the first term in curly brackets in (3.7) does contribute, so that in the limit $\vec{k} \to 0$ at arbitrary $\epsilon$ we obtain

$$R^\mu - L^\mu = -\frac{2g\overline{g}^2}{\omega_1 - \omega_2} C^\mu(q_2, q_1) \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{k}^{\,2})^\epsilon, \tag{3.12}$$

in agreement with [15, 16].

The real parts of (3.5), with account of (2.18), give

$$R^\mu + L^\mu = 2g \Big[ C^\mu(q_2, q_1) \left( 1 + \overline{g}^2 \bar{r} \right)$$

$$+\bar{g}^2\left\{-2t_1t_2(\mathcal{I}_5^\mu+\mathcal{L}_3^\mu)+2t_1\bar{r}_A\frac{p_A^\mu}{s_1}-2t_2\bar{r}_B\frac{p_B^\mu}{s_2}\right\}\Bigg] \ , \tag{3.13}$$

where $\mathcal{I}_5^\mu$ and $\mathcal{L}_3^\mu$ differ from the integrals $\mathcal{I}_5$ and $\mathcal{L}_3$ defined in (2.14) by the factors $(k_1-q_1)_\perp^\mu$ and $k_{1\perp}^\mu$, respectively, in the corresponding integrands,

$$\bar{r}=\left\{\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2}(\mathcal{I}_5-\mathcal{L}_3)-\vec{q}_2^{\,2}\mathcal{I}_{4B}+\frac{\Gamma^2(\epsilon)}{2\Gamma(2\epsilon)}(\vec{q}_1^{\,2})^\epsilon\left[\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{k}^2}\right)+\psi(1)+2\psi(\epsilon)-\psi(1-\epsilon)-2\psi(2\epsilon)\right.\right.$$
$$\left.\left.+\frac{1}{2(1+2\epsilon)(3+2\epsilon)}\left(\frac{\vec{q}_1^{\,2}+\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}(11+7\epsilon)+2\epsilon\frac{\vec{k}^2\vec{q}_1^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}-4\frac{\vec{k}^2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\right)\right]\right\}+\{A\longleftrightarrow B\}\ , \tag{3.14}$$

$$\bar{r}_A=-\vec{q}_1^{\,2}\vec{q}_2^{\,2}(\mathcal{I}_5-\mathcal{L}_3)+\vec{q}_2^{\,2}\mathcal{I}_{4B}+\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}_2^{\,2})^\epsilon\left(\ln\left(\frac{\vec{k}^2}{\vec{q}_2^{\,2}}\right)-\psi(1)-\psi(\epsilon)+\psi(1-\epsilon)+\psi(2\epsilon)\right)$$
$$-\frac{\Gamma^2(\epsilon)}{\vec{q}_1^{\,2}\Gamma(2\epsilon)}\frac{1}{2(1+2\epsilon)(3+2\epsilon)}\left\{\left[(\vec{q}_1^{\,2})^\epsilon\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}(11+7\epsilon)+\epsilon\vec{k}^2\vec{q}_1^{\,2}\frac{\vec{q}_1^{\,2}-\vec{k}^2}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\right.\right.\right.$$
$$\left.\left.\left.-\vec{k}^2\vec{q}_1^{\,2}\vec{q}_2^{\,2}\frac{\vec{q}_1^{\,2}+\vec{q}_2^{\,2}-2\vec{k}^2}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\right)\right]+[A\longleftrightarrow B]\right\}\ , \tag{3.15}$$

and

$$\bar{r}_B=\bar{r}_A(A\longleftrightarrow B). \tag{3.16}$$

As well as $R^\mu-L^\mu$, we can present $R^\mu+L^\mu$ in the explicitly gauge invariant form. It can be done using the real part of (2.25) to express $\mathcal{I}_5^\mu$ and $\mathcal{L}_3^\mu$ in terms of $\mathcal{I}_5$, $\mathcal{L}_3$ and $\mathcal{I}_{4A,B}$. We obtain:

$$R^\mu+L^\mu=2g\left\{C^\mu(q_2,q_1)+\bar{g}^2\left(\frac{(\vec{q}_1\vec{q}_2)C^\mu(q_2,q_1)+2\vec{q}_1^{\,2}\vec{q}_2^{\,2}\mathcal{P}^\mu}{2(\vec{q}_1^{\,2}\vec{q}_2^{\,2}-(\vec{q}_1\vec{q}_2)^2)}\left[\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}\vec{k}^2}{2}(\mathcal{I}_5-\mathcal{L}_3)\right.\right.\right.$$
$$-\vec{q}_2^{\,2}(\vec{q}_1\vec{k})\,\mathcal{I}_{4B}+\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}\left(\frac{(\vec{k}^2)^\epsilon}{2}(\vec{q}_1\vec{q}_2)\,(\psi(\epsilon)-\psi(1-\epsilon))+(\vec{q}_1^{\,2})^\epsilon(\vec{q}_2\vec{k})\left(\ln\left(\frac{\vec{k}^2}{\vec{q}_1^{\,2}}\right)-\psi(1)\right.\right.$$
$$\left.\left.\left.-\psi(\epsilon)+\psi(1-\epsilon)+\psi(2\epsilon)\right)\right)\right]+C^\mu(q_2,q_1)\left[-\frac{\vec{q}_2^{\,2}}{2}\mathcal{I}_{4B}+\frac{\Gamma^2(\epsilon)}{2\Gamma(2\epsilon)}\left(\frac{(\vec{k}^2)^\epsilon}{2}(\psi(\epsilon)-\psi(1-\epsilon))\right.\right.$$
$$+(\vec{q}_1^{\,2})^\epsilon\left[\psi(\epsilon)-\psi(2\epsilon)+\frac{1}{2(1+2\epsilon)(3+2\epsilon)}\left(\frac{\vec{q}_1^{\,2}+\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}(11+7\epsilon)+2\epsilon\frac{\vec{k}^2\vec{q}_1^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\right.\right.$$
$$\left.\left.\left.\left.-4\frac{\vec{k}^2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\right)\right]\right)\right]+\mathcal{P}^\mu\frac{\Gamma^2(\epsilon)(\vec{q}_1^{\,2})^\epsilon}{\Gamma(2\epsilon)(1+2\epsilon)(3+2\epsilon)}\left[\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}(11+7\epsilon)+\epsilon\vec{k}^2\vec{q}_1^{\,2}\frac{\vec{q}_1^{\,2}-\vec{k}^2}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\right.$$
$$\left.\left.\left.-\vec{k}^2\vec{q}_1^{\,2}\vec{q}_2^{\,2}\frac{\vec{q}_1^{\,2}+\vec{q}_2^{\,2}-2\vec{k}^2}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\right]\right)-\bar{g}^2\,(A\longleftrightarrow B)\right\}\ . \tag{3.17}$$

The same result is obtained from the real part of (3.5) if for $\mathcal{A}^\mu$ the representation (2.26) is used.

The integrals $\mathcal{I}_5$, $\mathcal{L}_3$ and $\mathcal{I}_{4A,B}$ defined in (2.14) cannot be written in terms of elementary functions at arbitrary $\epsilon$. For $\epsilon \to 0$ we have, with accuracy up to terms vanishing in this limit (see Appendix C):

$$\mathcal{I}_5 - \mathcal{L}_3 \simeq -\frac{1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\left[\frac{1}{\epsilon^2}\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^2}\right)^{\epsilon} - \frac{\pi^2}{6}\right]$$

$$+\frac{1}{\vec{q}_1^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon^2}\left(\frac{\vec{q}_1^{\,2}\vec{k}^2}{\vec{q}_2^{\,2}}\right)^{\epsilon} - \frac{\pi^2}{2} - 2L\left(1-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right] + \frac{1}{\vec{q}_2^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon^2}\left(\frac{\vec{q}_2^{\,2}\vec{k}^2}{\vec{q}_1^{\,2}}\right)^{\epsilon} - \frac{\pi^2}{2} - 2L\left(1-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right] , \tag{3.18}$$

$$\mathcal{I}_{4B} \simeq \frac{1}{\vec{q}_2^{\,2}}\left[\frac{(\vec{q}_2^{\,2})^{\epsilon} - 2(\vec{q}_1^{\,2})^{\epsilon}}{\epsilon^2} - \frac{\pi^2}{6} + 2L\left(1-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right] , \tag{3.19}$$

where

$$L(x) = \int_0^x \frac{dt}{t}\ln(1-t) . \tag{3.20}$$

Using these integrals, we obtain from (3.17)

$$R^\mu + L^\mu = 2g\Bigg\{C^\mu(q_2,q_1)\Bigg(1+\bar{g}^2\Bigg[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\ln(\vec{k}^2) - \frac{1}{2}\ln^2(\vec{k}^2) - \frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) + \frac{\pi^2}{2}$$

$$+\frac{\vec{k}^2}{3}\frac{\vec{q}_1^{\,2}+\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2} + \frac{1}{6}\left(11\frac{\vec{q}_1^{\,2}+\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}} - \frac{4\vec{k}^2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\right)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\Bigg]\Bigg)$$

$$+\mathcal{P}^\mu\bar{g}^2\frac{2}{3}\Bigg[\frac{\vec{k}^2}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\left(\vec{q}_1^{\,2}(\vec{q}_1^{\,2}-\vec{k}^2) + \vec{q}_2^{\,2}(\vec{q}_2^{\,2}-\vec{k}^2)\right) + \Bigg(11\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}-\vec{q}_2^{\,2}}$$

$$-\frac{\vec{k}^2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}(\vec{q}_1^{\,2}+\vec{q}_1^{\,2}-2\vec{k}^2)\Bigg)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\Bigg]\Bigg\} , \tag{3.21}$$

in agreement with [13], taking into account the difference in the energy scales (see [18]) and the charge renormalization (remind that in all formulae above the bare charge $g$ is used).

The limit $\vec{k} \to 0$ at arbitrary $D$ can be also considered without difficulties. The value of the integral $I_5$ in this limit is known [15]. In the $s$-channel physical region

$$I_5 \simeq \pi^{2+\epsilon}\Gamma(1-\epsilon)\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}\frac{(\vec{k}^2)^{\epsilon-1}}{s\vec{Q}^{\,2}}\left[\psi(\epsilon) - \psi(1-\epsilon) + i\pi\right] . \tag{3.22}$$

Therefore from (C.2) and (3.11) we obtain

$$\mathcal{I}_5 - \mathcal{L}_3 \simeq \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}\frac{(\vec{k}^2)^{\epsilon-1}}{\vec{Q}^{\,2}}\left[\psi(1-\epsilon) - \psi(\epsilon)\right] . \tag{3.23}$$

The integral $\mathcal{I}_{4A}$ (see (2.14)) is finite at $\vec{k} \to 0$ and can be calculated in this limit by standard methods. We have

$$\mathcal{I}_{4A} \simeq \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{Q}^2)^{\epsilon-1}\left[\psi(\epsilon) - \psi(2\epsilon)\right] . \tag{3.24}$$

Using (3.23) and (3.24), we obtain from (3.17)

$$R^\mu + L^\mu = 2gC^\mu(q_2,q_1)\left(1+\overline{g}^2\frac{\Gamma^2(\epsilon)}{2\Gamma(2\epsilon)}(\vec{k}^2)^\epsilon\left[\psi(\epsilon)-\psi(1-\epsilon)\right]\right), \tag{3.25}$$

that agrees with [15, 16], taking into account the difference in the energy scales (see [18]).

# 4 The one-gluon contribution to the BFKL kernel

The complicated analytical structure (3.1) of the RRG-vertex is irrelevant for the calculation of the contribution of the one-gluon production to the BFKL kernel at the NLO, where only the real parts of the production amplitudes do contribute, because only these parts interfere with the leading order amplitudes, which are real. Therefore, with the required accuracy, the production amplitude can be presented in the same form as in the leading order:

$$A_{2\to3} = 2s\,\Gamma^{c_1}_{A'A}\left(\frac{s_1}{\vec{k}^2}\right)^{\omega_1}\frac{1}{t_1}\,T^{c_1}_{A'A}\,\gamma^d_{c_1c_2}(q_1,q_2)\left(\frac{s_2}{\vec{k}^2}\right)^{\omega_2}\frac{1}{t_2}\,T^{c_2}_{B'B}\ , \tag{4.1}$$

where $\omega_i \equiv \omega(t_i)$, $\vec{k}^{\,2}$ is used as the energy scale and

$$\gamma^d_{c_1c_2}(q_1,q_2) = \frac{1}{2}\,T^d_{c_2c_1}\,e^*_\mu(k)\,(R^\mu+L^\mu)\ . \tag{4.2}$$

The contribution of the one-gluon production to the BFKL kernel for non-forward scattering with momentum transfer $q$ and irreducible representation $\mathcal{R}$ of the colour group in the $t$-channel is [18, 5]

$$\mathcal{K}^{G\,(\mathcal{R})}_{RRG}(\vec{q}_1,\vec{q}_2;\vec{q}) = \frac{1}{(2\pi)^{D-1}}\frac{\langle c_1c'_1|\hat{\mathcal{P}}_{\mathcal{R}}|c_2c'_2\rangle}{2n_{\mathcal{R}}}\sum_{d,\lambda}\gamma^d_{c_1c_2}(q_1,q_2)\left(\gamma^d_{c'_1c'_2}(q'_1,q'_2)\right)^*\ , \tag{4.3}$$

where $n_{\mathcal{R}}$ is the number of independent states in the representation $\mathcal{R}$, the sum is performed over colours and polarizations of the produced gluons and

$$q'_1 = q_1 - q\ , \qquad q'_2 = q_2 - q\ . \tag{4.4}$$

The most interesting representations $\mathcal{R}$ are the colour singlet (Pomeron channel) and the antisymmetric colour octet (gluon channel). We have for the singlet case

$$\langle c_1c'_1|\hat{\mathcal{P}}_1|c_2c'_2\rangle = \frac{\delta_{c_1c'_1}\delta_{c_2c'_2}}{N^2-1}\ , \qquad n_1 = 1\ , \tag{4.5}$$

and for the octet case

$$\langle c_1c'_1|\hat{\mathcal{P}}_8|c_2c'_2\rangle = \frac{f_{c_1c'_1c}f_{c_2c'_2c}}{N}\ , \qquad n_8 = N^2-1\ , \tag{4.6}$$

where $f_{abc}$ are the structure constants of the colour group. The vertex (4.2) is explicitly invariant under the gauge transformation

$$e^\mu(k) \to e^\mu(k) + k^\mu\chi\ , \tag{4.7}$$

so that we can use the relation

$$\sum_\lambda e_\mu^{*\,(\lambda)}(k) e_\nu^{(\lambda)}(k) = -g_{\mu\nu} \; . \tag{4.8}$$

Substituting (4.2) in (4.3), using (4.8) for the sum over polarizations and

$$\delta_{c_1 c_1'} \delta_{c_2 c_2'} T^d_{c_1 c_2} \left(T^d_{c_1' c_2'}\right)^* = N(N^2-1) \; , \qquad f_{c_1 c_1' c} f_{c_2 c_2' c} T^d_{c_1 c_2} \left(T^d_{c_1' c_2'}\right)^* = \frac{N^2(N^2-1)}{2} \tag{4.9}$$

for the sum over colour indices, we obtain with the NLO accuracy:

$$\mathcal{K}^{G\,(\mathcal{R})}_{RRG}(\vec q_1, \vec q_2; \vec q) = -\frac{c_{\mathcal{R}}}{8(2\pi)^{D-1}} 2g \left[ C_\mu(q_2', q_1')(R+L)^\mu + C_\mu(q_2, q_1)(R'+L')^\mu \right.$$

$$\left. -2g C_\mu(q_2, q_1) C^\mu(q_2', q_1') \right] \; , \tag{4.10}$$

where for the singlet ($\mathcal{R}=1$) and octet ($\mathcal{R}=8$) cases

$$c_1 = N \; , \qquad c_8 = \frac{N}{2} \; , \tag{4.11}$$

$C_\mu(q_2, q_1)$ is defined in (2.6), the sum $(R+L)^\mu$ is presented in two different forms in (3.13)-(3.16) and (3.17), $(R'+L')^\mu$ is obtained from $(R+L)^\mu$ by the substitution

$$\vec q_1 \to \vec q_1^{\,\prime} = \vec q_1 - \vec q \; , \qquad \vec q_2 \to \vec q_2^{\,\prime} = \vec q_2 - \vec q \, . \tag{4.12}$$

The convolution over the vector indices in (4.10) is easily performed with the help of the relations

$$C_\mu(q_2, q_1) C^\mu(q_2', q_1') = 2\left( \vec q^{\,2} - \frac{\vec q_1^{\,2} \vec q_2^{\,\prime\,2} + \vec q_1^{\,\prime\,2} \vec q_2^{\,2}}{\vec k^2} \right) \; ,$$

$$C_\mu(q_2, q_1) \mathcal{P}^\mu = \frac{\vec q_1^{\,2} + \vec q_2^{\,2}}{\vec k^2} - 1 \; . \tag{4.13}$$

In general case the convolution does not lead to noticeable simplifications, so that it has no sense to rewrite (4.10) in unfolded form. But at $\vec q = 0$ there is a huge simplification due to the equality

$$C_\mu(q_2, q_1) \left[ (\vec q_1 \vec q_2) C^\mu(q_2, q_1) + 2\vec q_1^{\,2} \vec q_2^{\,2} \mathcal{P}^\mu \right] = 0 \; . \tag{4.14}$$

For this case we obtain

$$\mathcal{K}^{G\,(\mathcal{R})}_{RRG}(\vec q_1, \vec q_2; \vec 0) = \frac{g^2 c_{\mathcal{R}}}{(2\pi)^{D-1}} \left\{ \frac{\vec q_1^{\,2} \vec q_2^{\,2}}{\vec k^2} \left[ 1 + 2\overline{g}^2 \left( -\vec q_2^{\,2} \mathcal{I}_{4B} + \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left( \frac{(\vec k^2)^\epsilon}{2} (\psi(\epsilon) - \psi(1-\epsilon)) \right.\right.\right.\right.$$

$$\left.\left.\left. + (\vec q_1^{\,2})^\epsilon (\psi(\epsilon) - \psi(2\epsilon)) \right)\right)\right] - \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \frac{\overline{g}^2 (\vec q_1^{\,2})^{\epsilon+1}}{(1+2\epsilon)(3+2\epsilon)} \left[ -12 \frac{\vec q_2^{\,2}}{\vec q_1^{\,2} - \vec q_2^{\,2}} \right.$$

$$\left.\left. + \frac{\vec k^2 \vec q_2^{\,2}}{(\vec q_1^{\,2} - \vec q_2^{\,2})^3} (3(\vec q_1^{\,2} + \vec q_2^{\,2}) - 2\vec k^2) + \epsilon \left( \frac{\vec q_1^{\,2} - 7\vec q_2^{\,2}}{(\vec q_1^{\,2} - \vec q_2^{\,2})} - \frac{\vec k^2}{(\vec q_1^{\,2} - \vec q_2^{\,2})^2} (2\vec q_1^{\,2} + \vec q_2^{\,2} - \vec k^2) \right) \right] \right\}$$

$$+\frac{g^2 c_{\mathcal{R}}}{(2\pi)^{D-1}}\left\{A \longleftrightarrow B\right\} . \tag{4.15}$$

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# A Appendix

In this Appendix we give the expressions of the integrals relevant for this paper, among those having the general form of Eqs. (2.13). The integrals with two or three denominators were already calculated in [13] at arbitrary $D$. Those with four and five denominators were calculated in Ref. [13] in the $\epsilon$-expansion. Unfortunately, it is not possible in general to express them in terms of elementary functions at arbitrary $D$. It is possible, however, to perform the integration over the longitudinal variables in the Sudakov decomposition for the integration variable $p$ and to express the result in terms of $(D-2)$-dimensional integrals over $\vec{p}$. As an illustration of the method, we will show in some details the steps involved in the calculation of $I_4^{(1)}$ for which it turns out that, in the multi-Regge kinematics (2.1), the complete integration can be performed. For the remaining integrals with four of five denominators, we will merely list the results.

Let us start from the integrals with two or three denominators:

$$I_2(q) = \frac{1}{i}\int d^Dp\,\frac{1}{(p^2+i\varepsilon)[(p+q)^2+i\varepsilon]} = -\frac{\pi^{2+\epsilon}\Gamma(1-\epsilon)}{2(1+2\epsilon)}\,\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}^{\,2})^\epsilon\ , \tag{A.1}$$

$$I_2^{\mu\nu}(q) = \frac{1}{i}\int d^Dp\,\frac{p^\mu p^\nu}{(p^2+i\varepsilon)[(p+q)^2+i\varepsilon]} = \frac{1}{4(3+2\epsilon)}\left[2(2+\epsilon)\,q^\mu q^\nu - q^2\,g^{\mu\nu}\right]I_2(q)\ , \tag{A.2}$$

$$I_3 = \frac{1}{i}\int d^Dp\,\frac{1}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+k)^2+i\varepsilon]} = \frac{1+2\epsilon}{\epsilon}\,\frac{I_2(q_2)-I_2(q_1)}{t_1-t_2}\ , \tag{A.3}$$

$$\begin{aligned} I_3^\mu &= \frac{1}{i}\int d^Dp\,\frac{p^\mu}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+k)^2+i\varepsilon]} = q_1^\mu\,\frac{I_2(q_1)-I_2(q_2)}{t_1-t_2} \\ &\quad + \frac{k^\mu}{(t_1-t_2)^2}\left[\frac{t_1}{\epsilon}I_2(q_1) + \left(t_2 - \frac{1+\epsilon}{\epsilon}\,t_1\right)I_2(q_2)\right]\ , \end{aligned} \tag{A.4}$$

$$\begin{aligned} I_3^{\mu\nu} &= \frac{1}{i}\int d^Dp\,\frac{p^\mu p^\nu}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+k)^2+i\varepsilon]} = \frac{g^{\mu\nu}}{4(1+\epsilon)}\,\frac{t_1 I_2(q_1)-t_2 I_2(q_2)}{t_1-t_2} \\ &\quad - \frac{q_1^\mu q_1^\nu}{2}\,\frac{I_2(q_1)-I_2(q_2)}{t_1-t_2} + \frac{k^\mu q_1^\nu + k^\nu q_1^\mu}{2(1+\epsilon)}\,\frac{-t_1 I_2(q_1) + [(1+\epsilon)\,t_1 - \epsilon\,t_2)]I_2(q_2)}{(t_1-t_2)^2} \\ &\quad + \frac{k^\mu k^\nu}{(t_1-t_2)^3}\left[-\frac{t_1^2 I_2(q_1)}{\epsilon(1+\epsilon)} + \left(\frac{t_1^2}{\epsilon} - \frac{t_1 t_2}{1+\epsilon} + \frac{(t_1-t_2)^2}{2}\right)I_2(q_2)\right]\ , \end{aligned} \tag{A.5}$$

$$\begin{aligned} I_3^{\mu\nu\rho} &= \frac{1}{i}\int d^Dp\,\frac{p^\mu p^\nu p^\rho}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+k)^2+i\varepsilon]} \\ &= \left(g^{\mu\nu}q_1^\rho + g^{\mu\rho}q_1^\nu + g^{\nu\rho}q_1^\mu\right)\frac{t_2 I_2(q_2) - t_1 I_2(q_1)}{4(3+2\epsilon)(t_1-t_2)} \\ &\quad - \left(g^{\mu\nu}k^\rho + g^{\mu\rho}k^\nu + g^{\nu\rho}k^\mu\right)\frac{t_1^2 I_2(q_1) - [(2+\epsilon)t_2(t_1-t_2)+t_2^2]I_2(q_2)}{4(1+\epsilon)(3+2\epsilon)(t_1-t_2)^2} \\ &\quad + q_1^\mu q_1^\nu q_1^\rho\,\frac{(2+\epsilon)\,[I_2(q_1)-I_2(q_2)]}{2(3+2\epsilon)(t_1-t_2)} \end{aligned}$$

$$+\Big(q_1^\mu q_1^\nu k^\rho + q_1^\mu k^\nu q_1^\rho + k^\mu q_1^\nu q_1^\rho\Big)\frac{t_1 I_2(q_1) - [t_2 + (1+\epsilon)(t_1 - t_2)]I_2(q_2)}{2(3+2\epsilon)(t_1-t_2)^2}$$

$$+\Big(q_1^\mu k^\nu k^\rho + k^\mu q_1^\nu k^\rho + k^\mu k^\nu q_1^\rho\Big)\frac{2t_1^2 I_2(q_1) - [2t_2^2 + (2+\epsilon)(t_1-t_2)(2t_2 + (1+\epsilon)(t_1-t_2))]I_2(q_2)}{2(1+\epsilon)(3+2\epsilon)(t_1-t_2)^3}$$

$$+k^\mu k^\nu k^\rho \Bigg\{ \frac{3[t_1^3 I_2(q_1) - t_2^3 I_2(q_2)]}{\epsilon(1+\epsilon)(3+2\epsilon)(t_1-t_2)^4}$$

$$-\left[\frac{3(3+\epsilon)t_2^2}{\epsilon(1+\epsilon)(3+2\epsilon)(t_1-t_2)^3} + \frac{3t_2}{(t_1-t_2)^2}\left(\frac{1}{2(3+2\epsilon)} + \frac{1}{\epsilon(1+\epsilon)}\right)\right.$$

$$\left.+\frac{1}{t_1-t_2}\left(-\frac{1}{4(3+2\epsilon)} + \frac{1}{\epsilon} + \frac{1}{4}\right)\right] I_2(q_2)\Bigg\} \ , \tag{A.6}$$

$$I_{3A} = \frac{1}{i}\int d^D p \, \frac{1}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+p_A)^2+i\varepsilon]} = -\frac{1+2\epsilon}{\epsilon}\frac{I_2(q_1)}{t_1} \ , \tag{A.7}$$

$$I_{3A}^\mu = \frac{1}{i}\int d^D p \, \frac{p^\mu}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+p_A)^2+i\varepsilon]} = \left(q_1^\mu + \frac{p_A^\mu}{\epsilon}\right)\frac{I_2(q_1)}{t_1} \ . \tag{A.8}$$

Let us consider now integrals with four or five denominators. As anticipated above, we will show in some detail how to calculate $I_4^{(1)}$ and simply give the final results for the remaining integrals.

The integral under consideration is

$$I_4^{(1)} = \frac{1}{i}\int d^D p \, \frac{1}{(p^2+i\varepsilon)[(p+q_1)^2+i\varepsilon][(p+p_A)^2+i\varepsilon][(p-p_B)^2+i\varepsilon]} \ . \tag{A.9}$$

The Sudakov decompositions for the integration momentum $p$ and for $q_1$ are

$$p = \beta p_A + \alpha p_B + p_\perp \ , \qquad q_1 \equiv p_A - p_{A'} = \frac{s_2}{s}p_A - \frac{\vec{q}_1^{\,2}}{s}p_B + q_{1\perp} \ . \tag{A.10}$$

After changing the integration variables to $\alpha$, $\beta$ and $p_\perp$ and using

$$d^D p = \frac{s}{2}\, d\alpha\, d\beta\, d^{D-2}p \ , \tag{A.11}$$

we perform the integration over the variable $\alpha$ by the method of residues. This leads in the multi-Regge kinematics to the following result

$$I_4^{(1)} = \pi\int_0^1 d\beta\beta^2 \int \frac{d^{D-2}p}{\vec{p}^{\,2}(\vec{p}+\beta\vec{q}_1)^2[\vec{p}^{\,2} + \beta(1-\beta)(-s-i\varepsilon)]}$$

$$-\pi\frac{s_2}{s}\int_0^1 d\beta\beta^2 \int \frac{d^{D-2}p}{\vec{p}^{\,2}[(\vec{p}+\beta\vec{q}_1)^2 + \beta(1-\beta)\vec{q}_1^{\,2}][(\vec{p}+\beta\vec{q}_1)^2 + \beta(1-\beta)(-s_2-i\varepsilon)]} \ . \tag{A.12}$$

The first integral in the R.H.S. of the above equation can be manipulated as follows:

$$\left[I_4^{(1)}\right]_1 \equiv \pi\int_0^1 d\beta\beta^2 \int \frac{d^{D-2}p}{\vec{p}^{\,2}(\vec{p}+\beta\vec{q}_1)^2[\vec{p}^{\,2} + \beta(1-\beta)(-s-i\varepsilon)]}$$

$$= \pi \int \frac{d^{D-2}p}{\vec{p}^{\,2}(\vec{p}+\vec{q}_1)^2} \int_0^1 d\beta \frac{(1-\beta)^{D-5}}{(1-\beta)\vec{p}^{\,2}+\beta(-s-i\varepsilon)}$$

$$\simeq \pi \int \frac{d^{D-2}p}{\vec{p}^{\,2}(\vec{p}+\vec{q}_1)^2} \int_0^1 d\beta \frac{(1-\beta)^{D-5}}{\vec{p}^{\,2}+\beta(-s-i\varepsilon)} \;, \tag{A.13}$$

where the first equality follows from the change of variables $\vec{p} \to \beta\vec{p}$ and $\beta \to 1-\beta$, while the last approximated equality holds since in the $s \to \infty$ limit $s + \vec{p}^{\,2} = s$. Then, the integral over $\beta$ can be performed:

$$\int_0^1 d\beta \frac{(1-\beta)^{D-5}}{\delta+\beta} = \int_0^1 d\beta \frac{(1-\beta)^{D-5}-1}{\delta+\beta} + \int_0^1 \frac{d\beta}{\delta+\beta}$$

$$\simeq \int_0^1 d\beta \frac{(1-\beta)^{D-5}-1}{\beta} + \int_0^1 \frac{d\beta}{\delta+\beta} \simeq \psi(1) - \psi(D-4) + \ln\left(\frac{1}{\delta}\right) \;, \tag{A.14}$$

where $\delta \equiv \vec{p}^{\,2}/(-s-i\varepsilon)$ is a quantity tending to zero and $\psi(x)$ is the logarithmic derivative of $\Gamma(x)$. After performing the integration over $p_\perp$, we obtain for $\left[I_4^{(1)}\right]_1$ the following result:

$$\left[I_4^{(1)}\right]_1 = \frac{\pi^{2+\epsilon}\Gamma(1-\epsilon)}{s\,t_1} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}_1^{\,2})^\epsilon \left[\ln\left(\frac{-s-i\varepsilon}{-t_1}\right) + \psi(1-\epsilon) - \psi(\epsilon)\right] \;. \tag{A.15}$$

In the last expression, the terms $-i\varepsilon$ which appears in the argument of the logarithm fixes the prescription for the analytic continuation of this function to the region of positive $s$. In the following, this term will always be omitted in the final results, but it should be understood to accompany $-s$, $-s_1$ or $-s_2$ every time these quantities appear in the argument of the logarithm. The second integral in the R.H.S. of Eq. (A.12) can be decomposed as follows:

$$\left[I_4^{(1)}\right]_2 = -\pi\frac{s_2}{s}\int_0^1 d\beta\beta^2 \int \frac{d^{D-2}p}{(\vec{p}+\beta\vec{q}_1)^2[\vec{p}^{\,2}+\beta(1-\beta)\vec{q}_1^{\,2}][\vec{p}^{\,2}+\beta(1-\beta)(-s_2-i\varepsilon)]}$$

$$= -\pi\frac{\vec{q}_1^{\,2}}{s}\int_0^1 d\beta\beta^2 \int \frac{d^{D-2}p}{\vec{p}^{\,2}(\vec{p}+\beta\vec{q}_1)^2[\vec{p}^{\,2}+\beta(1-\beta)\vec{q}_1^{\,2}]}$$

$$-\pi\frac{s_2}{s}\int_0^1 d\beta\beta^2 \int \frac{d^{D-2}p}{\vec{p}^{\,2}(\vec{p}+\beta\vec{q}_1)^2[\vec{p}^{\,2}+\beta(1-\beta)(-s_2-i\varepsilon)]} \;, \tag{A.16}$$

where it has been used that in the multi-Regge kinematics $s_2 \gg \vec{q}_1^{\,2}$. The second integral in the R.H.S. of the above equation has the same structure as $\left[I_4^{(1)}\right]_1$ (see Eq. (A.13)) and does not need to be calculated. The first one can be easily evaluated by the usual Feynman parametrization technique. The final result for $\left[I_4^{(1)}\right]_2$ is then

$$\left[I_4^{(1)}\right]_2 = \frac{\pi^{2+\epsilon}\Gamma(1-\epsilon)}{s\,t_1} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}_1^{\,2})^\epsilon \left[\ln\left(\frac{-t_1}{-s_2}\right) + \psi(1) - \psi(1-\epsilon)\right] \;. \tag{A.17}$$

Combining Eqs. (A.15) and (A.17), we obtain the final result for $I_4^{(1)}$:

$$I_4^{(1)} = \frac{\pi^{2+\epsilon}\Gamma(1-\epsilon)}{s\,t_1} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}_1^{\,2})^\epsilon \left[\ln\left(\frac{-s}{-s_2}\right) + \psi(1) - \psi(\epsilon)\right] \;. \tag{A.18}$$

The same procedure described for the calculation of $I_4^{(1)}$ can be applied for the integrals $I_{4A}$, $I_5$ and $I_5^\mu$, except that for them the integrations in the transverse space cannot be performed in complete way. We list here the final results:

$$I_{4A} = -\frac{\pi^{2+\epsilon}\Gamma(1-\epsilon)}{s_1}\left[\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}_1^{\,2})^{\epsilon-1}\left(\ln\left(\frac{-s_1}{-t_1}\right)+\psi(1-\epsilon)-2\psi(\epsilon)+\psi(2\epsilon)\right)+\mathcal{I}_{4A}\right], \tag{A.19}$$

$$I_5 = \frac{\pi^{2+\epsilon}\Gamma(1-\epsilon)}{s}\left[\ln\left(\frac{(-s)\vec{k}^2}{(-s_1)(-s_2)}\right)\mathcal{I}_3+\mathcal{L}_3-\mathcal{I}_5\right], \tag{A.20}$$

$$I_5^\mu = q_1^\mu I_5 - p_A^\mu\frac{I_{4A}}{s}+p_B^\mu\frac{I_{4B}}{s}+\frac{\pi^{2+\epsilon}\Gamma(1-\epsilon)}{s}\left[-\ln\left(\frac{(-s)\vec{k}^2}{(-s_1)(-s_2)}\right)\mathcal{I}_3^\mu-\mathcal{L}_3^\mu-\mathcal{I}_5^\mu\right], \tag{A.21}$$

where

$$\mathcal{I}_{4A}=\int_0^1\frac{dx}{1-x}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\left[\frac{x}{[(1-x)\vec{k}_1^2+x(\vec{k}_1+\vec{q}_2)^2](\vec{k}_1-x\vec{k})^2}-\frac{1}{(\vec{k}_1+\vec{q}_2)^2(\vec{k}_1-\vec{k})^2}\right],$$

$$\mathcal{I}_5 = \int_0^1\frac{dx}{1-x}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{k}_1^2[(1-x)\vec{k}_1^2+x(\vec{k}_1-\vec{q}_1)^2]}\left(\frac{x^2}{(\vec{k}_1-x\vec{k})^2}-\frac{1}{(\vec{k}_1-\vec{k})^2}\right),$$

$$\mathcal{I}_5^\mu = \int_0^1\frac{dx}{1-x}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{(k_1-q_1)_\perp^\mu}{\vec{k}_1^2[(1-x)\vec{k}_1^2+x(\vec{k}_1-\vec{q}_1)^2]}\left(\frac{x^2}{(\vec{k}_1-x\vec{k})^2}-\frac{1}{(\vec{k}_1-\vec{k})^2}\right),$$

$$\mathcal{L}_3 = \int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{k}_1^2(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_2)^2}\ln\left(\frac{(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_2)^2}{\vec{k}^2\,\vec{k}_1^2}\right),$$

$$\mathcal{L}_3^\mu = \int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{k_{1\perp}^{\ \mu}}{\vec{k}_1^2(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_2)^2}\ln\left(\frac{(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_2)^2}{\vec{k}^2\,\vec{k}_1^2}\right),$$

$$\mathcal{I}_3 = \int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{\vec{k}_1^2(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_2)^2},$$

$$\mathcal{I}_3^\mu = \int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{k_{1\perp}^{\ \mu}}{\vec{k}_1^2(\vec{k}_1-\vec{q}_1)^2(\vec{k}_1-\vec{q}_2)^2}, \tag{A.22}$$

$I_{4B}$ in the expression for $I_5^\mu$ (Eq. (A.21)) is obtained from $I_{4A}$ by the replacements $p_A \longleftrightarrow p_B$ and $q_1 \longleftrightarrow -q_2$. In the multi-Regge kinematics these replacements imply that $I_{4B} = I_{4A}(s_1 \to s_2, \vec{q}_1 \longleftrightarrow -\vec{q}_2)$.

# B Appendix

In this Appendix we derive the relations (2.21) and (2.22). Let us consider first Eq. (2.21). From the definition of $\mathcal{F}_5^\mu$ given in the first of Eqs. (2.16) and from the expression for $I_5^\mu$ given in Eq. (A.21), we obtain

$$\mathcal{F}_5^\mu = \frac{s}{\pi^{2+\epsilon}\Gamma(1-\epsilon)}\left[q_1^\mu I_5 - p_A^\mu\frac{I_{4A}}{s}+p_B^\mu\frac{I_{4B}}{s}-I_5^\mu\right]+\frac{1}{2}\ln\left(\frac{s(-s_1)(-s_2)}{(-s)s_1s_2}\right)\mathcal{I}_3^\mu\,. \tag{B.1}$$

If we contract both sides of the above equation with $2k_\mu$, we get

$$
\begin{aligned}
2k_\mu \mathcal{F}_5^\mu &= \frac{s}{\pi^{2+\epsilon}\Gamma(1-\epsilon)} \Bigg[ (t_1 - t_2) I_5 - \frac{s_1}{s} I_{4A} + \frac{s_2}{s} I_{4B} + I_4^{(1)} - I_4^{(2)} \\
&+ \frac{1}{2} \ln\left( \frac{s(-s_1)(-s_2)}{(-s)s_1 s_2} \right) \left[ \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left( (\vec{q}_2^{\,2})^{\epsilon-1} - (\vec{q}_1^{\,2})^{\epsilon-1} \right) + (t_1 - t_2)\mathcal{I}_3 \right] \Bigg] ,
\end{aligned} \tag{B.2}
$$

where we have used the relations

$$
k_\mu I_5^\mu = \frac{I_4^{(2)} - I_4^{(1)}}{2} , \qquad k_\mu \mathcal{I}_3^\mu = \frac{1}{2} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left( (\vec{q}_2^{\,2})^{\epsilon-1} - (\vec{q}_1^{\,2})^{\epsilon-1} \right) + \frac{t_1 - t_2}{2} \mathcal{I}_3 . \tag{B.3}
$$

$I_4^{(2)}$ in Eq. (B.2) is obtained from $I_4^{(1)}$ by the replacements $p_A \longleftrightarrow p_B$ and $q_1 \longleftrightarrow -q_2$. In the multi-Regge kinematics these replacements imply $I_4^{(2)} = I_4^{(1)}(t_1 \to t_2, s_2 \to s_1)$. Using Eqs. (A.18)-(A.20) and the third of Eqs. (2.16) in the R.H.S. of Eq. (B.2), it is easy to obtain the relation (2.21).

Let us consider now the relation (2.22). Starting again from Eq. (B.1) and contracting both sides with $2(q_1+q_2)_\mu$, we get

$$
\begin{aligned}
2(q_1+q_2)_\mu \mathcal{F}_5^\mu &= \frac{s}{\pi^{2+\epsilon}\Gamma(1-\epsilon)} \Bigg[ (t_1 + t_2) I_5 + \frac{s_1}{s} I_{4A} + \frac{s_2}{s} I_{4B} - I_4^{(1)} - I_4^{(2)} + 2I_4 \\
&+ \frac{1}{2} \ln\left( \frac{s(-s_1)(-s_2)}{(-s)s_1 s_2} \right) \left[ \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left( (\vec{q}_1^{\,2})^{\epsilon-1} + (\vec{q}_2^{\,2})^{\epsilon-1} - 2(\vec{k}^2)^{\epsilon-1} \right) + (t_1 + t_2)\mathcal{I}_3 \right] \Bigg] ,
\end{aligned} \tag{B.4}
$$

where we have used the relations

$$
\begin{aligned}
(q_1+q_2)_\mu I_5^\mu &= \frac{I_4^{(1)} + I_4^{(2)}}{2} - I_4 + t_1 I_5 , \\
(q_1+q_2)_\mu \mathcal{I}_3^\mu &= \frac{1}{2} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left( (\vec{q}_1^{\,2})^{\epsilon-1} + (\vec{q}_2^{\,2})^{\epsilon-1} - 2(\vec{k}^2)^{\epsilon-1} \right) + \frac{t_1 + t_2}{2} \mathcal{I}_3 .
\end{aligned} \tag{B.5}
$$

In the previous expressions a new integral appeared

$$
I_4 = \frac{1}{i} \int d^D p \, \frac{1}{[(p+q_1)^2 + i\varepsilon][(p+q_2)^2 + i\varepsilon][(p+p_A)^2 + i\varepsilon][(p-p_B)^2 + i\varepsilon]} . \tag{B.6}
$$

It can be evaluated using the procedure illustrated in Appendix A for the integral $I_4^{(1)}$. The result is

$$
I_4 = -\frac{\pi^{2+\epsilon}\Gamma(1-\epsilon)}{s} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{k}^2)^{\epsilon-1} \left[ \ln\left( \frac{(-s)\vec{k}^2}{(-s_1)(-s_2)} \right) + \psi(\epsilon) - \psi(1-\epsilon) \right] . \tag{B.7}
$$

Using Eqs. (A.18)-(A.20), (B.7) and the third of Eqs. (2.16) in the R.H.S. of Eq. (B.4), it is easy to obtain the relation (2.22).

# C Appendix

In this Appendix we consider the $\epsilon \to 0$ limit for the integrals $\mathcal{I}_3$, $\mathcal{I}_{4A}$ and for the combination $\mathcal{I}_5 - \mathcal{L}_3$. The definitions of these integrals are given in (2.14). The integral $\mathcal{I}_{4B}$, whose expression for $\epsilon \to 0$ is given in Eq. (3.19), can be obtained from $\mathcal{I}_{4A}$ by the replacement $\vec{q}_1 \longleftrightarrow -\vec{q}_2$.

Let us start from $\mathcal{I}_3$. This integral was calculated in the Appendix II of Ref. [13] to the order $\epsilon^0$. We report here its expression

$$\mathcal{I}_3 = \frac{1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\left[\frac{1}{\epsilon}+\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^2}\right)\right] + \frac{1}{\vec{q}_1^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon}+\ln\left(\frac{\vec{q}_1^{\,2}\vec{k}^2}{\vec{q}_2^{\,2}}\right)\right] + \frac{1}{\vec{q}_2^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon}+\ln\left(\frac{\vec{q}_2^{\,2}\vec{k}^2}{\vec{q}_1^{\,2}}\right)\right]$$
$$\simeq \frac{1}{\epsilon}\left[\frac{(\vec{q}_1^{\,2}\vec{q}_2^{\,2})^{\epsilon-1}}{(\vec{k}^2)^{\epsilon}} + \frac{(\vec{q}_1^{\,2}\vec{k}^2)^{\epsilon-1}}{(\vec{q}_2^{\,2})^{\epsilon}} + \frac{(\vec{q}_2^{\,2}\vec{k}^2)^{\epsilon-1}}{(\vec{q}_1^{\,2})^{\epsilon}}\right], \tag{C.1}$$

where the last approximated equality holds with accuracy $O(\epsilon)$.

The combination of integrals $\mathcal{I}_5 - \mathcal{L}_3$ can be expressed in terms of the $I_5$ and $\mathcal{I}_3$, according to Eq. (A.20):

$$\mathcal{I}_5 - \mathcal{L}_3 = -\frac{s}{\pi^{2+\epsilon}\,\Gamma(1-\epsilon)}\, I_5 + \ln\left(\frac{(-s)\vec{k}^2}{(-s_1)(-s_2)}\right)\mathcal{I}_3 \,. \tag{C.2}$$

The integral $I_5$ was calculated in the Appendix III of Ref. [13] to the order $\epsilon^0$:

$$\begin{aligned}
&- \frac{s}{\pi^{2+\epsilon}\,\Gamma(1-\epsilon)}\, I_5 = \\
&- \frac{1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{(-s)\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(-s_1)(-s_2)}\right)+\frac{1}{2}\ln^2\left(\frac{(-s)\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(-s_1)(-s_2)}\right)+\frac{\pi^2}{3}\right] \\
&+ \frac{1}{\vec{q}_1^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{(-s_1)(-s_2)\vec{q}_1^{\,2}}{(-s)\vec{q}_2^{\,2}}\right)+\frac{1}{2}\ln^2\left(\frac{(-s_1)(-s_2)\vec{q}_1^{\,2}}{(-s)\vec{q}_2^{\,2}}\right)-2L\left(1-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right] \\
&+ \frac{1}{\vec{q}_2^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{(-s_1)(-s_2)\vec{q}_2^{\,2}}{(-s)\vec{q}_1^{\,2}}\right)+\frac{1}{2}\ln^2\left(\frac{(-s_1)(-s_2)\vec{q}_2^{\,2}}{(-s)\vec{q}_1^{\,2}}\right)-2L\left(1-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right],
\end{aligned} \tag{C.3}$$

where

$$L(x) = \int_0^x \frac{dt}{t}\ln(1-t)\,. \tag{C.4}$$

Using in (C.2) the expression for $\mathcal{I}_3$ given in the first line of Eq. (C.1), we get

$$\begin{aligned}
\mathcal{I}_5 - \mathcal{L}_3 = &- \frac{1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^2}\right)+\frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^2}\right)-\frac{\pi^2}{6}\right] \\
&+ \frac{1}{\vec{q}_1^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{\vec{q}_1^{\,2}\vec{k}^2}{\vec{q}_2^{\,2}}\right)+\frac{1}{2}\ln^2\left(\frac{\vec{q}_1^{\,2}\vec{k}^2}{\vec{q}_2^{\,2}}\right)-\frac{\pi^2}{2}-2L\left(1-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right] \\
&+ \frac{1}{\vec{q}_2^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{\vec{q}_2^{\,2}\vec{k}^2}{\vec{q}_1^{\,2}}\right)+\frac{1}{2}\ln^2\left(\frac{\vec{q}_2^{\,2}\vec{k}^2}{\vec{q}_1^{\,2}}\right)-\frac{\pi^2}{2}-2L\left(1-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right]
\end{aligned}$$

$$\simeq -\frac{1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\left[\frac{1}{\epsilon^2}\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^2}\right)^{\epsilon}-\frac{\pi^2}{6}\right]+\frac{1}{\vec{q}_1^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon^2}\left(\frac{\vec{q}_1^{\,2}\vec{k}^2}{\vec{q}_2^{\,2}}\right)^{\epsilon}-\frac{\pi^2}{2}-2L\left(1-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right]$$

$$+\frac{1}{\vec{q}_2^{\,2}\vec{k}^2}\left[\frac{1}{\epsilon^2}\left(\frac{\vec{q}_2^{\,2}\vec{k}^2}{\vec{q}_1^{\,2}}\right)^{\epsilon}-\frac{\pi^2}{2}-2L\left(1-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right]\ , \tag{C.5}$$

where the last approximated equality holds with accuracy $O(\epsilon)$ and we have used $\ln(-s)=\ln s - i\pi$ and the analogous relations for $s_1$ and $s_2$, valid in the $s$-channel physical region.

Finally, let us consider $\mathcal{I}_{4A}$. According to (A.19), it can be written in the following way:

$$\mathcal{I}_{4A}=-\frac{s_1}{\pi^{2+\epsilon}\,\Gamma(1-\epsilon)}\,I_{4A}-\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}(\vec{q}_1^{\,2})^{\epsilon-1}\left[\ln\left(\frac{(-s_1)}{\vec{q}_1^{\,2}}\right)+\psi(1-\epsilon)-2\psi(\epsilon)+\psi(2\epsilon)\right]\ . \tag{C.6}$$

The integral $I_{4A}$ was calculated in the Appendix III of Ref. [13] to the order $\epsilon^0$:

$$-\frac{s_1}{\pi^{2+\epsilon}\,\Gamma(1-\epsilon)}\,I_{4A}=\frac{1}{\vec{q}_1^{\,2}}\left[\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\ln\left(\frac{(-s_1)\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-\ln^2\vec{q}_2^{\,2}+2\ln\vec{q}_1^{\,2}\ln(-s_1)\right.$$

$$\left.-\pi^2+2L\left(1-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right]\ . \tag{C.7}$$

Using this expression in the R.H.S. of Eq. (C.6) together with the expansion to the order $\epsilon^0$ of the second term in the R.H.S. of the same equation, we get

$$\mathcal{I}_{4A}=\frac{1}{\vec{q}_1^{\,2}}\left[-\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\vec{q}_1^{\,2}+\frac{1}{2}\ln^2\vec{q}_1^{\,2}-\frac{2}{\epsilon}\ln\vec{q}_2^{\,2}-\ln^2\vec{q}_2^{\,2}-\frac{\pi^2}{6}+2L\left(1-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right]$$

$$\simeq\frac{1}{\vec{q}_1^{\,2}}\left[\frac{(\vec{q}_1^{\,2})^{\epsilon}-2(\vec{q}_2^{\,2})^{\epsilon}}{\epsilon^2}-\frac{\pi^2}{6}+2L\left(1-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right]\ , \tag{C.8}$$

where the last approximated equality holds with accuracy $O(\epsilon)$.

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